Analysis on the Quantitative Evaluation Method of University Students’ Comprehensive Ability

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Abstract: The quantitative evaluation of university students’ comprehensive ability is an issue pertaining to multi-objective group decision making. Taking into consideration the relative weights of decision-makers’ evaluation level is conducive to eliminate the influence of deviations of such level on the comprehensive evaluation. This paper conducts a comparison analysis on the methods of fuzzy mathematics, grey correlation, set pair analysis and attribute mathematics, which can better realize multi-objective quantitative analysis.

Keywords: University students, Comprehensive ability, Quantitative evaluation, Method

INTRODUCTION

The evaluation of university students’ comprehensive ability represents an important part in educational evaluation. Many factors affect the evaluation of university students’ comprehensive ability, from quantitative to qualitative, all have an obvious hierarchical relationship. Yet these factors which restrict and influence each other, exert quite different degrees of influence on the object. Correct analysis of the inherent logical law among the factors and establishment of a reasonable mathematical model are vital to the quantitative evaluation of comprehensive ability.

DETERMINATION OF RELATIVE WEIGHT OF INFLUENCING FACTORS AND DECISION MAKERS

The evaluation of students’ comprehensive ability pertains to the issue of quantitative analysis of multi-objective groups, and the hierarchical chart of the evaluation system is shown in Figure 1.

![Hierarchical Chart](image)

**Figure 1** Hierarchical Chart

**Determination of the initial weight value of the index of each layer**

The application of analytic hierarchy process (AHP) to determine the weight of each influencing factor on the target can help make the judgment more scientific and accurate. When the weight of each indicator is determined by analytic hierarchy process (AHP), taking two indicators $X_i$ and $X_j$ at a time, use $D_{ij}$ as the effects ratio of $X_i$ and $X_j$, form the pairwise comparison matrix $D = (D_{ij})_{nm}$, and use approximate calculation method to determine the maximum eigenvalue $\lambda$ and weight vector $\omega$ of the matrix $D$. If the inconsistency of $D$ is acceptable after check, then $\omega$ is the weight allocation subset. If the inconsistency of $D$ is unacceptable, the $D_{ij}$ value in the pairwise comparison matrix $D$ must be adjusted to make the inconsistency of $D$ acceptable. The value of $D_{ij}$ are determined by experts according to the scale of “1-9”.

**Determination of the relative weight of the evaluation level of decision makers**

All kinds of evaluators who engage in the evaluation of students’ comprehensive ability may have different influence on the evaluation. Therefore, the relative weight...
of decision makers should be considered in group decision-making.

A total of $T$ decision makers, when evaluating the importance of $m$ indexes according to a certain criterion, adopt the analytic hierarchy process to determine the weight $\omega_i = (\omega_{i1}, \omega_{i2}, \omega_{i3}, \ldots, \omega_{im})$ ($t = 1, 2, \ldots, T$) ($j = 1, 2, \ldots, m$) of decision makers to each index. Based on the concept of “minimum variance” in classical mathematics, and different levels between the actual weight $\omega_{ij}$ and the optimal weight $w_j$ ($w_j$ takes the average value of $\omega_{ij}$) of decision makers’ evaluation, the relative weights of decision makers are determined, and the relative weight of the evaluation level of the $t$th decision maker is:

$$A_t = \frac{1}{C_t} \sum_{i=1}^{t} C_i$$

$C_t$ is the proportion of the relative evaluation weight error in the total weight error caused by the $t$th decision maker, thus $C_t$ is:

$$C_t = \left[ \frac{\sum_{j=1}^{m} (\omega_{ij} - w_j)^2}{\sum_{j=1}^{m} (\omega_{ij} - w_j)^2} \right]^{1/2}$$

**Determination of multi-index weight considering the level of decision makers**

Taking into consideration the evaluation level of $T$ decision makers, the relative weights of $m$ indexes obtained by weighted geometric mean method are as follows:

$$\omega_i = \frac{1}{T} \prod_{t=1}^{T} \omega_{it}^{A_t}$$

$t = 1, 2, \ldots, T$; $j = 1, 2, \ldots, m$

**Final weight of each index**

From the hierarchical structure model shown in Figure 1, the importance weight $\omega_{X_i} (i = 1, 2, \ldots, n)$ of the $i$th criterion of the criterion layer ($X$) relative to the target layer ($Y$) and the single sort weight $\omega_{X_{ij}}$ of the index layer ($X_{ij}$) relative to the $i$th criterion at the $j$th layer after the evaluation level of the decision maker is taken into account can be obtained by the above method. Taking the index weight which is not controlled by a certain criterion as 0, the weight $A_y$ of the index layer $X_y$ relative to the target layer $X$ can be obtained by using the synthetic weight calculation method:

$$A_y = \omega_{X_y} \cdot \omega_{X}$$

$i = 1, 2, \ldots, n$; $j = 1, 2, \ldots, m$

**DETERMINATION OF EVALUATION INDEX VALUE**

The evaluation value of the quantitative index is determined according to the magnitude of the index value according to the prior regulations. The qualitative indexes can be scored according to the interval value, and the latter can be determined by the set-valued statistical method.

If there is $n$ criteria in the evaluation criterion layer, and the $t$th criterion has $m$ indexes, then the evaluation index value of the $t$th appraiser is:

$$b = \frac{1}{T} \sum_{i=1}^{T} \omega_i$$

of which $b_{ij} = (i = 1, 2, \ldots, n; j = 1, 2, \ldots, m; k = 1, 2, \ldots, l)$ is the evaluation value of the $j$th indicator of the $k$th appraiser to be evaluated under the $i$th evaluation criterion.

If the weight of the $t$th appraiser is $A_t = (t = 1, 2, \ldots, T)$, then the set-value statistical evaluation value $b_{ij}$ of the $j$th index under the $i$th criterion for the $k$th appraiser is:

$$b_{ij} = \frac{1}{T} \sum_{t=1}^{T} A_t (b_{ij}^{(t)} - b_{ij}^{(0)})^2$$

$A_t = \frac{1}{T} \sum_{i=1}^{T} A_i (b_{ij}^{(t)} - b_{ij}^{(0)})^2$

**ANALYSIS ON QUANTITATIVE EVALUATION MODEL**

**Fuzzy comprehensive evaluation method**

The fuzzy comprehensive evaluation method applies the fuzzy transformation principle and the maximum membership transformation principle in fuzzy mathematics, and establishes the fuzzy comprehensive evaluation model by taking into account the factors related to the object to be evaluated. When fuzzy mathematics is used to evaluate, mathematical models $M(\land, \lor)$ and $M(+, +)$ are often used. The evaluation results of the principal element deterministic model $M(\land, \lor)$ are determined by the largest index, and the changes of other indexes in a certain range are not considered comprehensively, which affects the scientific rationality of the comprehensive evaluation results; $M(+, +)$ operator gives consideration to all evaluation indicators by the weight, thus is applicable to the requirements of the overall indicators, and the same to the evaluation of university students’ comprehensive ability. If $M(+, +)$ is adopted, and the weight distribution $Fuzzy$ set of the index set is $A_y$, the membership function values of each level are constructed according to the evaluation set of each index, and the evaluation matrix $R$ is obtained, then the comprehensive evaluation result $B = A_y \cdot R$ is matrix multiplication.

**Grey correlation evaluation method**

When evaluate the quality of multiple objects with the grey correlation theory, if there is $m$ evaluation indexes,
take the ideal evaluation value of each index of the optimal objects as the reference sequence \( X_a = \{ X_a(j) \mid (j = 1, 2, \ldots, m) \} \), and the evaluation value corresponding to each index of concrete objects to be evaluated as the comparison sequence, then the comparison sequence of the \( k \) th evaluation index is \( X_i = \{ X_i(j) \mid (j = 1, 2, \ldots, m) \} \). Based on this, the correlation degree \( \gamma \) between the objects to be evaluated and the optimal objects can be obtained. The greater the degree of correlation, the greater the degree of association between the evaluated object and the optimal object. According to the degree of correlation, not only the relatively optimal object can be selected, but also the difference between the evaluated object and the optimal object can be reflected.

The correlation coefficient \( \varepsilon_i(p) \) between the comparison sequence \( X_i \) and the reference sequence \( X_a \) at the time \( p \) is:

\[
\varepsilon_i(p) = \frac{\Delta_{\text{min}} + \rho \Delta_{\text{max}}}{\Delta_{\text{obs}}(p) + \rho \Delta_{\text{max}}}
\]

among which \( \Delta_{\text{min}} = \min \min_j |X_i(j) - X_a(j)| \) is the minimum difference and \( \Delta_{\text{max}} = \max \max_j |X_i(j) - X_a(j)| \) is the maximum difference between the two poles; \( \Delta_{\text{obs}}(p) = |X_i(j) - X_a(j)| \) is the absolute difference between two sequences at \( p \) time; \( \rho \) is resolution ratio, \( \rho \in [0, 1] \), usually \( \rho = 0.5 \).

The degree of correlation between the comparison sequence \( X_i \) and the reference sequence \( X_a \) can be compared with the correlation \( \gamma_i \).

\[
\gamma_i = \frac{1}{m} \sum_{j=1}^{m} A_j \varepsilon_i(p)
\]

Wherein \( A_j \) is the index weight. The greater the value of correlation \( \gamma_i \), the greater the correlation between the comparison sequence \( X_i \) and the reference sequence \( X_a \). According to the degree of correlation \( \gamma_i \), we can impose order on each object to be evaluated based on its advantages and disadvantages. The same fundamental unit and common intersection point are a must in calculating the correlation coefficient. When the fundamental units of the data sequence are different, the initialization process should be carried out.

**Set pair analysis evaluation**

In set pair analysis, the identical degree is the ratio of the number of characteristics \( N \) shared by two sets to the total number of characteristics \( L \) of the two sets, which is denoted as \( d = \frac{N}{L} \). Set pair analysis evaluation method uses the concept of the identical degree of two sets in set pair analysis to obtain that of the index to be evaluated and the ideal index. It calculates the identical degree of the system to be evaluated and the ideal system by combining the weight of each index, and determines the order of the evaluation objects according to the value of the identical degree. A system to be evaluated and an ideal system are formed into a set pair in set pair analysis evaluation. The ideal system is composed of the optimal value of each index in the \( L \) systems to be evaluated. If the original identical degree between the evaluation value \( X_j(k) \) of the \( j \) th index in the \( k \) th system \( C_k \) to be evaluated and the optimal value \( X_j(0) \) of the corresponding index in the ideal system \( C_0 \) is \( a^{(k)} \), then after considering the weight of each index \( A_j \), the new identical degree \( a^{(k)} \) should be

\[
a^{(k)} = \sum_{j=1}^{m} A_j a_{jk} \quad (k = 1, 2, \ldots, L)
\]

The order of quality of the \( L \) systems to be evaluated is determined by the value of \( a^{(k)} \), that is, the larger \( a^{(k)} \) is, the better the system is.

**Attribute comprehensive evaluation method**

Attribute comprehensive evaluation generally carries out the single index attribute measure analysis first, then the multi-index attribute measure analysis, and finally the discriminatory analysis. The single index attribute measure analysis determines the attribute measure function according to the relationship between the index value and evaluation categories, and calculates the attribute measure value based on the eigenvalues of each index; In multi-index comprehensive attribute measure analysis, the comprehensive attribute measure is obtained by weighted summation of single-index attribute measures; Discriminatory analysis offers a recognition criteria based on the results of comprehensive attribute measure analysis to identify which evaluation category it belongs to. For the ordered evaluation class, the confidence criterion is adopted.

Assuming the evaluation set of each index is \( B = (B_1, B_2, \ldots, B_L, B_p) \), the constructible attribute measure is \( \mu \), when single index attribute measure analysis is conducted, the \( k \) th appraisee’s evaluation value of the \( j \) th index under the \( i \) th criterion is \( a_{i,j} \), “\( a_{i,j} \in B_r \) (1 \leq r \leq R)" means “\( a_{i,j} \) belongs to the \( r \) th class \( B_r \)”, and its attribute measure is \( \mu_{ij} = \mu(a_{i,j} \in B_r) \). \( \mu_{ij} \) should be limited to \( \mu_{ij} \geq 0, \sum_{r=1}^{R} \mu_{ij} = 1 \).

In the multi-index comprehensive attribute measure analysis, the multi-index comprehensive attribute measure of the \( k \) th appraisee can be obtained from the index weight \( A_j \) and the single-index attribute measure \( \mu_{ij} \):

\[
\mu_{ij} = \sum_{j=1}^{m} \sum_{r=1}^{R} A_j \mu_{ij} \quad (r = 1, 2, \ldots, L, R)
\]

The comprehensive evaluation and ranking of the appraisees can be carried out by applying the confidence.
criterion in the comprehensive attribute measure $\mu^k_r$. Set the degree of confidence is $\lambda$ (generally between 0.6 and 0.7). If: $r_0 = \min \{ \sum_{i=1}^{r} \mu^k_i \geq \lambda, 1 \leq r \leq R \}$, then the $k$th appraisee is considered to be belonged to category $B_{r_0}$.

**Comparison of the four models**

Four mathematical models for the quantitative evaluation of university students’ comprehensive ability are analyzed in this paper. Generally speaking, fuzzy comprehensive evaluation method lacks, to a certain extent, systematic reference standards, so it falls short in analyzing the dynamic correlation degree among various factors, which may lead to evaluation errors. The grey system is a system whose internal characteristics are known in part, and in which the relationship between the internal factors and factors is not completely clear. The grey system theory puts forward the method of correlation analysis, that is, the degree of correlation among the factors can be measured according to the similarity or difference of the development trend of the influencing factors, which can reveal the characteristics and degree of dynamic correlation of things. The set pair analysis method does not involve the concepts of membership function in fuzzy mathematics theory and correlation degree in grey relation theory. The whole evaluation process is clear, simple and easy to understand and master. The qualitative description of things is called attribute. Attribute comprehensive evaluation takes attributes as a set in terms of thinking, establishes an attribute measure space, characterizes the properties of things with the distribution of attribute set measure in attribute space, and realizes evaluation through attribute measure analysis and recognition.

Fuzzy mathematics and attribute mathematics comprehensive evaluation build the evaluation set to calculate the comprehensive evaluation value of the system to be evaluated; While set pair analysis and grey correlation comprehensive evaluation establish ideal system set as a reference to impose order on the system to be evaluated based on its quality.

**CONCLUSION**

Quantitative evaluation of university students’ comprehensive ability is an issue of multi-objective group decision making. The relative weight of decision-maker’s level can be determined by the concept of “minimum variance” in mathematics, and the weight and evaluation value of each influencing factor can be obtained by the analytic hierarchy process and sets value statistics method after developing the qualitative and quantitative rules for assigning the influencing factors respectively, taking into account the relative weight of decision-maker’s level. In a nutshell, fuzzy mathematics, grey correlation, set pair analysis and attribute mathematics can all be used, attending to different needs, to evaluate quantitatively the comprehensive ability of college students.

**REFERENCES**


