The Research on Carbon Emission Forecast Based on GM (1, 1) Model in China

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Abstract: The carbon emission of China has become the focus of the world, and China's future carbon emissions forecasting will help achieve the emission reduction target of 2030. The carbon emission data in China from 2000 to 2018 were selected, and the short-term prediction was made based on Grey Model GM (1, 1) model. The model test results show that the relative error is level 2, the correlation degree is level 4, and mean variance ratio and small error probability are both level 1, which shows that the predicted results has good forecasting performance. By 2025, China's carbon emission will exceed 41 tons of carbon, and the carbon dioxide emission situation during the "14th five-year plan" is still severe.

Keywords: Carbon emission, GM (1, 1), Forecast.

INTRODUCTION

At the end of 2015, the Paris climate conference was successfully held, and the issue of carbon emission once again aroused the focus of discussion among participants around the world. The Paris agreement, adopted at the climate conference, calls for the world to achieve zero emissions by the second half of the 21st century. As a major emitter of carbon dioxide, the Chinese government has set its own action target, which is to reach the peak of national carbon dioxide emission around 2030. Therefore, accurate prediction of China's future carbon emissions is of great significance.

Grey Model has great advantages in the prediction field [Tianxiang Yao et.al. 2009]. It does not need too many samples and good distribution rules, requires little computation and has strong adaptability, hence it is widely used in many fields [Ming Xie et.al. 2009]. Therefore, GM(1,1) model is applied in this paper to predict China's future carbon emissions, providing a research basis for realizing the emission reduction target in 2030.

THE FORMATION OF A HOLISTIC VIEW OF KNOWLEDGE

The grey model was put forward by professor Deng Julong, a famous scholar in China. Professor Deng found that a large number of information systems are uncertain systems, that is, the information in the system is partly known and partly unknown. Grey system theory can deal with uncertain information by generating part of the known information, extracts valuable information from uncertain system, and realizes the correct understanding of the whole system. In previous studies of grey model, the original data were directly used to build the model, but sometimes the regularity of the original data was not strong, such as the existence of outliers, which would affect the effect of the model. The grey prediction model is to process the original data and establish the differential equation with the newly generated data. The generated data can enhance the regularity of the data and eliminate outliers, so that the results obtained by the model are more accurate [Zheng-Xin Wang et.al. 2018].

GM (1,1) model is the most commonly used grey model, which consists of a first order differential equation containing only one variable.

GM (1,1) prediction

The specific steps are as follows

1) The original time series be denoted as
   \[ x^{(0)} = \{x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n)\} \]

2) Sum up the original time series and generate a sequence \(1-AGO\) as
   \[ x^{(1)} = [x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n)] \]

3) Equation (2) is used to form the following first-order differential equation
   \[ \frac{dx^{(1)}}{dt} + ax^{(1)} = u \]

\(a\) is development parameter and \(u\) is control parameter.

4) Use the least square method to solve the parameters and \(\mu\):

\[
A = (B^T B)^{-1} B^T Y, \quad Y = \left[ \begin{array}{c} x^{(0)}(2) \\ \vdots \\ x^{(0)}(n) \end{array} \right]
\]

\[
B = \left[ \begin{array}{cccc} 1 & x^{(0)}(2) & \vdots & x^{(0)}(n) \\ \frac{1}{2} [x^{(0)}(1) + x^{(0)}(2)] & 1 & \vdots & \vdots \\ \frac{1}{2} [x^{(0)}(2) + x^{(0)}(3)] & \vdots & \ddots & \vdots \\ \frac{1}{2} [x^{(0)}(n-1) + x^{(0)}(n)] & \vdots & \ddots & 1 \end{array} \right]
\]
(5) The time response function of the model can be calculated as follows:
\[ \hat{x}^{(1)}(k + 1) = \left( \hat{x}^{(0)}(1) - \frac{\hat{u}}{\hat{a}} \right) e^{-\hat{a}k} + \frac{\hat{u}}{\hat{a}} \quad (k = 0, 1, 2, \ldots) \]

(6) By reducing the above sequence, the grey prediction model of \( X(0) \) can be obtained as follows:
\[ \hat{x}^{(0)}(k + 1) = \hat{x}^{(1)}(k + 1) - \hat{x}^{(1)}(k) = (1 - e^\delta) \left( \hat{x}^{(0)}(1) - \frac{\hat{u}}{\hat{a}} \right) e^{-\hat{a}k} \quad (k = 0, 1, 2, \ldots) \]

**GM (1,1) test**

(1) Residual examination
\( X^{(0)}(i) \) is the original sequence, and \( \hat{X}^{(0)}(i) \) is the prediction sequence.

Absolute error \( e(i) = X^{(0)}(i) - \hat{X}^{(0)}(i) \)

Relative error \( \varepsilon(i) = e(i) / X^{(0)}(i) \)

(2) Correlation test
Correlation coefficient
\[ \eta(i) = \frac{\min \left| e(i) \right| + \delta \cdot \max \left| e(i) \right|}{e(i) + \delta \cdot \max \left| e(i) \right|} \]

Correlation degree
\[ \gamma = \frac{1}{n} \sum \eta(i) \]

\( \delta \) is resolution ratio which is set in [0, 1], and \( \delta = 0.5 \) is selected in this paper.

(3) Posterior variance test
The mean value of the original sequence
\[ \bar{X}^{(0)} = \frac{1}{n} \sum X^{(0)}(i) \]

The standard deviation of the original sequence
\[ S_x = \sqrt{\frac{1}{n-1} \sum (X^{(0)}(i) - \bar{X}^{(0)})^2} \]

The mean value of absolute error
\[ \bar{e} = \frac{1}{n} \sum e(i) \]

The standard deviation of absolute error
\[ S_e = \sqrt{\frac{1}{n-1} \sum (e(i) - \bar{e})^2} \]

Mean variance ratio
\[ C = \frac{S_x}{S_e} \]

Small error probability
\[ P = P \left| e(i) - \bar{e} < 0.6745 \cdot S_e \right| \]

The above three methods all judge the accuracy of GM(1,1) by investigating the residual error. Among them, the smaller the relative error and the mean variance ratio are, the better; the larger the correlation degree and the small error probability are, the better [Xiangyan Zeng et al. 2019].

The accuracy grade is shown in Table 1 for reference of the test model. In general, the most commonly used test index is relative error.

<table>
<thead>
<tr>
<th>Accuracy class</th>
<th>Relative error</th>
<th>Correlation degree</th>
<th>Mean variance ratio</th>
<th>Small error probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level 1</td>
<td>0.01</td>
<td>0.9</td>
<td>0.35</td>
<td>0.95</td>
</tr>
<tr>
<td>Level 2</td>
<td>0.05</td>
<td>0.8</td>
<td>0.5</td>
<td>0.8</td>
</tr>
<tr>
<td>Level 3</td>
<td>0.1</td>
<td>0.7</td>
<td>0.65</td>
<td>0.7</td>
</tr>
<tr>
<td>Level 4</td>
<td>0.2</td>
<td>0.6</td>
<td>0.8</td>
<td>0.6</td>
</tr>
</tbody>
</table>

**EMPIRICAL RESEARCH**

**Data collection**

At present, China has not published carbon emissions, so it is necessary to estimate China’s carbon emissions. Hence, this article collected China’s coal consumption, oil consumption and gas consumption (Unit: ten thousand tons of standard coal) from 2000 to 2018 which are shown in Table 2, and the carbon emission coefficient for all kinds of energy (Unit: ton of carbon/ten thousand tons of standard coal) determined by national development and reform commission (NDRC) energy research institute is selected to calculate the carbon emission which are shown in Table 3. The calculation results of carbon emissions (Unit: ton of carbon) from 2000 to 2018 in China are shown in Table 2.
Table 2 the raw data of energy consumption and carbon emission

<table>
<thead>
<tr>
<th>Year</th>
<th>Coal consumption</th>
<th>Oil consumption</th>
<th>Gas consumption</th>
<th>Carbon emission</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>100670.34</td>
<td>32332.08</td>
<td>3233.21</td>
<td>95528.51</td>
</tr>
<tr>
<td>2001</td>
<td>105771.96</td>
<td>32975.96</td>
<td>3733.13</td>
<td>99939.26</td>
</tr>
<tr>
<td>2002</td>
<td>116160.25</td>
<td>35611.17</td>
<td>3900.27</td>
<td>109314.68</td>
</tr>
<tr>
<td>2003</td>
<td>138352.27</td>
<td>39613.68</td>
<td>4532.91</td>
<td>128517.47</td>
</tr>
<tr>
<td>2004</td>
<td>161657.26</td>
<td>45825.92</td>
<td>5296.46</td>
<td>149897.55</td>
</tr>
<tr>
<td>2005</td>
<td>189231.16</td>
<td>46523.68</td>
<td>6272.86</td>
<td>171351.27</td>
</tr>
<tr>
<td>2006</td>
<td>207402.11</td>
<td>50131.73</td>
<td>7734.61</td>
<td>187685.85</td>
</tr>
<tr>
<td>2007</td>
<td>225795.45</td>
<td>52945.14</td>
<td>9343.26</td>
<td>203788.96</td>
</tr>
<tr>
<td>2008</td>
<td>229236.87</td>
<td>53542.04</td>
<td>10900.77</td>
<td>207400.21</td>
</tr>
<tr>
<td>2009</td>
<td>240666.22</td>
<td>55124.66</td>
<td>11764.41</td>
<td>217249.70</td>
</tr>
<tr>
<td>2010</td>
<td>249568.42</td>
<td>62752.75</td>
<td>14425.92</td>
<td>229528.72</td>
</tr>
<tr>
<td>2011</td>
<td>271704.19</td>
<td>65023.22</td>
<td>17803.98</td>
<td>248898.14</td>
</tr>
<tr>
<td>2012</td>
<td>275464.53</td>
<td>68363.46</td>
<td>19302.62</td>
<td>254319.71</td>
</tr>
<tr>
<td>2013</td>
<td>280999.36</td>
<td>71292.12</td>
<td>22096.39</td>
<td>261402.53</td>
</tr>
<tr>
<td>2014</td>
<td>279328.74</td>
<td>74090.24</td>
<td>24270.94</td>
<td>262747.89</td>
</tr>
<tr>
<td>2015</td>
<td>273849.49</td>
<td>78672.62</td>
<td>25364.4</td>
<td>261805.79</td>
</tr>
<tr>
<td>2016</td>
<td>270320</td>
<td>79788</td>
<td>27904</td>
<td>260943.17</td>
</tr>
<tr>
<td>2017</td>
<td>270911.58</td>
<td>84323.47</td>
<td>31397.04</td>
<td>265576.51</td>
</tr>
<tr>
<td>2018</td>
<td>273760</td>
<td>87696</td>
<td>36192</td>
<td>271797.05</td>
</tr>
</tbody>
</table>

Table 3 Carbon emissions coefficient of different energy

<table>
<thead>
<tr>
<th>Energy</th>
<th>Coal</th>
<th>Oil</th>
<th>Natural gas</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coefficient</td>
<td>0.7476</td>
<td>0.5825</td>
<td>0.4435</td>
</tr>
</tbody>
</table>

GM(1, 1) prediction

In this paper, the carbon emission data in China from 2000 to 2018 were taken as the original sequence, and the GM (1, 1) model was established by MATLAB, so as to predict the carbon emission in the future. The MATLAB procedure of GM (1, 1) is as follows:

\[
x_0 = [95528.51 \ 99939.26 \ 109314.68 \ 128517.47 \\
149897.55 \ 171351.27 \ 187685.85 \ 203788.96 \\
207400.21 \ 217249.7 \ 229528.72 \ 248898.14 \ 254319.71 \\
261402.53 \ 262747.89 \ 261805.79 \ 260943.17 \\
265576.51 \ 271797.05];
\]

\[
x_1 = \text{cumsum}(x_0);
\]

\[
b = [-0.5*[x_1(1:18)+x_1(2:19)] \ \text{ones}(18,1)\]
\]

\[
y = x_0(2:19);
\]

\[
a = (b'*b)^(-1)*b'*y
\]

\[
k = 0;
\]

\[
x(k+1) = (x_0(1)-a(2)/a(1))^\exp(-a(1)*k) + a(2)/a(1)
\]

\[
k = 1:18;
\]

\[
x(k+1) = (1-\exp(a(1)))^*(x_0(1)-a(2)/a(1))^\exp(-a(1)*k)
\]

\[
jdwc = x_0' - x
\]

\[
xdwc = jdwc ./ x_0'
\]
glx=(min(abs(jdwc))+0.5*max(abs(jdwc)))/abs(jdwc)+0.5*max(abs(jdwc)))
gld=sum(glx)/19
ysjz=mean(x0)
ysbzc=sum((x0-ysjz).^2)/19
jdwcjz=mean(jdwc)
jdwcjz=sum((jdwc-jdwcjz).^2)/19
c=jdwcjz/ysbzc
s0=0.6745*ysbzc;
year1=2000:2018
plot(year1,x0,'o');
hold on;
year2=2000:2025
k=19:25;
x(k+1)=(1-exp(a(1)))*(x0(1)-a(2)/a(1))*exp(-a(1)*k)
plot(year2,x,*);
xlabel('Year');
ylabel('Carbon Emission / ton of carbon')
legend('Actual data','Forecasted data')
The prediction results of GM (1,1) is illustrated by Figure 1.

| Table 4  The results of the test index and accuracy class |
|-----------------|---------------- |-----------------|-----------------|-----------------|
| Index           | Relative error | Correlation degree | Mean variance ratio | Small error probability |
| Value           | -0.03          | 0.60              | 0.13              | 1                |
| Accuracy class  | Level 2        | Level 4           | Level 1           | Level 1          |

**CONCLUSIONS**

GM(1,1) model is applied to predict China's future carbon emission. The accuracy class of the prediction result reaches level 2, indicating that GM(1,1) is feasible for carbon emission prediction in the short and medium term with high accuracy. China's carbon emissions will exceed 41 tons of carbon by 2025, and carbon emission can be reduced by improving energy efficiency, adjusting industrial structure and reducing fossil energy consumption.

**REFERENCES**

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